SYNOPSIS

All matter consists of four elements, which are solid, liquid, gas and plasma. The general name of last three states liquid, gas and plasma is termed as 'fluid 'and hence clearly most of the matters in the universe are in fluid state. In a scientific language, a fluid is a substance that deforms continuously under the application of a shear stress no matter how small the shear stress may be. According to the concept of viscosity, fluid is divided into two categories: Real fluid and Ideal fluid. All fluids which are having viscosity are known as real. An ideal fluid is supposed to have no viscosity, surface tension and compressibility.

Fluid Mechanics is a study of fluids which deals with the behavior of fluids at rest and in motion. In recent times, fluid mechanics finds its application almost in all fields of Engineering, Astrophysics, Biology, Metrology, Physics, Chemistry, Geophysics etc. It is easy to appreciate the principles of fluid mechanics when we observes a few natural phenomena like the flight of birds in air, the motion of fish in water and circulation of blood in human body.

Fluids are of two types: Newtonian in behavior and non–Newtonian. The fluid for which the relationship between shearing stress and the rate of deformation is linear is said to be Newtonian fluid. And a fluid that does not obey Newton's law of viscosity is said to be non – Newtonian fluid. In non–Newtonian fluids, shearing stress and rate of deformation are not linearly related. These types of fluids are found to be of great commercial importance. They are handled extensively by chemical industries namely plastics and polymers. Biological and biomedical devices make use of the rheological aspect of non – Newtonian fluids. The wide usage and application of these fluids in the diverse industrial fields have prompted modern researchers to explore extensively the field of non – Newtonian fluids.

There are several types of non –Newtonian fluid models of which are proposed by scientists working in this area. Several empirical models are generally used to approximate the experimental data are available for such fluids. It may not be possible to list all such models exhaustively. However few of them are presented by Williamson ( ), Skelland ( ) and Kapur ( )

A well–structured flow characterized by smooth and organized motion of fluid particles in laminae or well defined layers, usually occurring at relatively small Reynold number is called Laminar. And an unstable / agitated flow characterized by unorganized motion of fluid particles in random direction, usually occurring at higher Reynold number is said to be turbulent flow. In other words, viscous flow regimes are classified as laminar or turbulent on the basics of flow

structure. In the laminar regime, flow structure is characterized by smooth motion in laminae or layers. Flow structure in the turbulent regime is characterized by random, three–dimensional motions of fluid particles in addition to the mean motion.

Literally, fluids are classified as liquids or gases, which may be electrically conducting such as mercury or ionized gas. The electrical conductivity of the fluid plays, its role, only when, fluid is moving in an electromagnetic field, otherwise the fluid may be regarded as an ordinary fluid (non–electrically conducting fluid). Magneto fluid dynamics (MFD) is the study of the motion of an electrically conducting fluid in the presence of a magnetic field. When the fluid is incompressible, such as liquid mercury, and its other properties such as viscosity, thermal conductivity and electrical conductivity are regarded as constant the word magneto hydrodynamics (MHD) or hydromagnetics is used.

In two-dimensional boundary layer flows all flow parameters are independent of the third coordinate. Presently this type of flow geometry is considered almost exclusively. In three – dimensional flow processes, in contrast, the outer potential flow is a function of two space coordinates parallel to the wall surrounded by the flow, whereas the boundary layer shows all the velocity components which, in the most general case, are dependent on all three space coordinates.

The mathematical modeling of the most of physical phenomena (including fluid flow problems) leads to the differential equations. These equations may be either partial differential equations or ordinary differential equations. To study a real physical situation one needs to solve these equations. Since solution of differential equations yields to some functions of independent variables it is often hard to solve these equations. Infect there is no full proof method available even to solve a differential equation which looks quite simple.

The method to search particular type of transformations which transforms the partial differential equation into ordinary differential equation is known as similarity method. That is the term “Similarity transformation” of a partial differential equation shall be defined to be a transformation of independent and dependent variables occurring in the equation such that the number of independent variables appearing in the transformed equation is at least on less than in the original equation. If a partial differential equation has two independent variables, a similarity transformation would transform the equation into an ordinary differential equation.

The notable contribution in development similarity parameters for physical sciences was made by Reynolds (1895), Blasius (1908), Buckingham (1914) and Sedov (1942) who have put similarity technique in a more meaningful way till Morgan (1952), following the concepts of Birkhoff (1950), showed that the boundary layer equations were invariant under certain group of transformations which reduced the number of independent variables by one. The power of various similarity techniques is recently demonstrated by various authors like Hansen (1964), Moran (Thesis, 1968) Seshadri and Na (1985), Rogers and Ames (1989), Bluman (Thesis), Bluman and Kumei (1989), Bluman and Anco (2002) and Drachner (1996). There are four widely and frequently used similarity methods are found in literature. These methods are: Free Parameter method, Separation of Variables method, Group Theoretic method and Dimensional Analysis technique.

Among all these similarity methods, the group theoretic method is more attractive as it is derived systematically from most general group symmetry (invariance) law. The number of symmetries of partial differential equation yields the number of similarity solutions.

The applications of various similarity techniques based on different type of group transformations are found in literature.

PRESENT INVESTIGATION:

In present thesis, using group transformation techniques, Similarity solutions for both two-dimensional and three-dimensional, incompressible, laminar boundary layer flows for non – Newtonian fluids of different models are derived and discussed in detail. Finite Lie group method namely one parameter and two parameter Scaling group transformation techniques are used to solve boundary layer equations for non – Newtonian fluids. Similarity solutions for three-dimensional MHD boundary layer flow of non – Newtonian fluids are also discussed.

The entire research work is divided into **SIX** chapters:

As the succession of the various chapters presented, **Chapter – 1** is chosen in order to build up a strong structure in logical manner to provide knowledge of very basic concepts of fluid mechanics and magneto hydrodynamics. It is the essential part of study to have better understanding of behavior of particle in flow. This chapter deals with definitions and a brief discussion of Newtonian and non – Newtonian fluids and also contains some useful information

about the rheological development of different non – Newtonian fluid models. Out of these different Non-Newtonian fluid models, the Visco-in-elastic fluids are receiving our attention due to special form of functional relation between their stress-strain components. These Visco-in-elastic fluids can be classified into two categories: fluids for which shearing stress is explicit function of rate of strain. (The well-known power law fluid will be found in this category) and the fluids for which shearing stress is implicit function of rate of strain. These fluids are mathematically more complex. (The Powell-Eyring fluids, Williamson fluids, Prandtl fluids etc. are found in this category). We mainly confine on Non-Newtonian Visco-in-elastic fluids in which shearing stress is implicit function of rate of strain. In the sub sequent chapters, we consider detail study of both two-dimensional and Three-dimensional flow these fluids past various surfaces.

In **Chapter-2**, basic definition of group and group transformations is given. Out of various similarity techniques available in literature, the similarity techniques based on group theory are quite powerful tool to determine similarity transformation. In this chapter we have also proposed the generalized group of transformations, so-called finite Lie group transformation, to transform non-linear Partial differential equations of BVP type in to ordinary differential equations. The present proposed generalized group transformation includes other group transformation techniques as its special cases. Experience revolves that the group theory methods, have produced powerful theorems and yield results with a minimum of mathematical busy-work. On the other hand, the group theory methods are difficult for the average engineer to follow it because their motivation is mathematical application and not physical/theoretical development and hence this has inhibited their wide use. The application of present proposed method is supported by proper counter examples.

Chapter-3 is devoted to the brief survey of different numerical methods. Out of these methods, the method of satisfaction of asymptotic boundary condition (MSABC) that was developed by Nascheim et al (1965) after successive research papers on integration. This method is non-iterative and found quite promising to derive find numerical solutions of non-linear boundary layer equations. Further the similarity solution for the non-linear partial differential equations governing the laminar, incompressible boundary layer flow of class of non-Newtonian fluid over a wedge of an angle θ is derived through generalized group method that discussed in Chapter-2. The approximate solution for the said similarity equation through present method of

satisfaction of asymptotic boundary condition (MSABC) is obtained with necessary modification in this technique. The effect of variation of different flow parameters on flow field of two non-Newtonian fluid models, namely, Power-law and Powell-Eyring fluids is also studied. It is observed that, for both fluids the shear–stress at the wall decreases and velocity of the fluids is increase. is increased.

The natural convection incompressible boundary layer flows of Non-Newtonian Visco-in-elastic fluids past vertical plate are discussed in **Chapter 4.** A generalized group method discussed in Chapter-2 is used to derive similarity equations of the basic governing equation that are highly non-linear coupled partial differential equations. The similarity solutions for different Non-Newtonian, Visco-in-elastic fluids like Williamson fluids; Powell-Eyring fluids, Prandtl-Eyring fluids etc are derived. Using the numerical technique of Nachtsheim et al, discussed in Chapter- is modified and extended to find non-iterative solution of the present coupled non-linear differential equation for both low Prandtl number and high Prandtl number cases. It is observed that for the large value of Prandtl number some fluid models have similar behavior. The computer program for the solution of the present set of non-linear equations along with one sample output is given at the end of this chapter. A process in which a stream imposes fluid flow or it is imposed by forcing a fluid through passage by providing pressure difference is called forced convection flow.

Two-point boundary value problems occur in all branches of engineering and science. In these problems the boundary conditions are specified at two points. To complicate the matter, the governing differential equations for a majority of such problems are nonlinear; since analytic solutions do not in general exist, solutions have to be obtained by numerical methods. One could tackle this type of problem easily if it could get transformed in to initial value problem.

In Chapter-5, the theory of transformation of linear boundary value problems to initial value problems is discussed. The generalized group method presented in chapter-2 is applied to find the missing initial condition. This method is known as initial value method. As an application of theory, the boundary layer equations governing the two-dimensional jet flow with variable fluid properties is considered for both dissipative and non-dissipative jets. After a similarity transformation is made, the boundary layer equations are reduced to a nonlinear, third-order, ordinary differential equation with two boundary conditions given at the surface of the plate and the third at infinity. The present method of transformation is introduced the entire

boundary value problem with homogeneous boundary conditions is first transformed in to a problem with non homogeneous boundary conditions and subsequently entire BVP is transformed to an initial value problem. The successful application of this type of transformations in Blasius' equation makes one wonder whether it can be extended to other equations. The initial value problems are solving numerically. The present initial value method certainly eliminates the trial-and-error method as trial-and-error method usually takes a long computing time and the solution frequently is very sensitive to even a very small change of the assumed initial condition.

In **Chapter-6**, the system of partial differential equations governing the steady, laminar, incompressible three dimensional boundary layer flow of the class of non-Newtonian visco-in-elastic fluids are derived. The multiparameter (for present case-two-parameter) group-theoretic method is employed to derive the similarity transformations for present flow equation. The purpose of similarity transformations is to transform the governing system of highly non-linear partial differential equations along with auxiliary conditions to system of ordinary differential equation known as similarity equations.

The numerical solutions using non-iterative asymptotic boundary condition technique is obtained for Prandtl-Eyring fluids, Williamson fluids using classic Runge Kutta method with MATLAB ODE SOLVER in dimensionless form. Mutual comparisons of the velocity and temperature distribution of these fluids are interesting.

In conclusion, we would like to say that we have attempted success to extend the different similarity techniques to simplify different flows geometries. The numerical data presented in this thesis are hopefully useful to those who are working with chemical engineering and engineering boundary value problems.

Research scholar Research Guide

(M. Sadique) (Dr. M.G Timol)